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# PRACTICAL OPTIONS FOR THE RE-DEFINITION OF THE NIGERIAN VERTICAL REFERENCE SYSTEM: A CASE STUDY OF LAGOS STATE

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***ABSTRACT***
*Adopting a height system, especially in areas with a spatially-vast land mass, is a complicated choice between physically and geometrically meaningful systems, given the level of computational and observational rigor involved in either case. For Nigeria, there is no clear legislation as to the height system that should be used by practitioners, and this uncertainty about the adopted height system for the country has resulted in an untenable situation in which multiple height systems are used across the country. The aim of this study is to evaluate practical options for redefining and harmonizing the Nigerian Vertical Reference System (VRS), using the lowlands of Lagos State as a case study. The approach involved the acquisition of spirit-leveled elevation differences, GNSS ellipsoidal heights, and absolute gravity observations on 218 benchmarks across the state, and the comparative computation of three height candidates: (1) Helmert Orthometric Heights (HOH) obtained by applying the orthometric correction to spirit-leveled elevation differences using measured gravity; (2) Normal Orthometric Heights (NOH) obtained by applying the normal orthometric correction to spirit-leveled elevation differences using normal gravity; and (3) HOH derived from GNSS ellipsoidal heights using the South-Western Gravimetric Geoid 2018 (SWGG'018). Statistical analysis of the three options using a student's t-test of significant difference at the 95% confidence level indicates that there is no significant difference in the results obtained by all three options within the study area. However, considering the observational convenience and computational simplicity, option 3 is recommended for the redefinition of the Nigerian VRS. The study further recommends that a national height system be legislated for use by all practitioners in the production of topographic plans.*
***Keywords: Orthometric Height System, Ellipsoidal Height System, Gravity Field, Geopotential Number, Spheropotential Number.***

## 1.0 INTRODUCTION

It is a common belief that water flows from high points to low points. For this reason, flood control procedure requires that built-up settlements are located physically above the flood plain. Vertical Reference Systems (VRS) are broadly classified into two types: physical systems, which depend on the Earth's gravity field measured along the plumb line, and geometrical systems, which are independent of the gravity field (Brown, 2016). However, the common belief of water flowing downhill is only true when the heights are represented by a physical height system (Yilmaz, 2008). This is because the flow of fluids is governed by the force of gravity and not merely elevation difference (Featherstone and Kuhn, 2006). Although realized by a Vertical Reference Frame (VRF), a VRS defines the origin and the orientation of fundamental planes or axes for a set of measured heights. It also includes the underlying and fundamental mathematical and physical models upon which the heights are determined (Seeber, 2003). Therefore, a Reference system could simply be described as a set of parameters and idealized theoretical descriptions/model for an intended real-world positioning system. The Orthometric height is an example of a physical height system while Normal Heights, Normal Orthometric Heights, Ellipsoidal Heights and Dynamic Heights are examples of the geometrical height systems (Fotopoulos, 2003). The Nigerian VRF consists of a network of 250 geodetic leveling lines covering a total distance of over 20,000km (Isioye ***et al***, 2010). With its origin at Apapa datum, the Nigerian leveling network stretches across the country with the aim of providing unified height system across the entire nation realized through established fundamental and standard benchmarks (FBM's and SBM's) whose measures of internal accuracy were accurately computed to international standard (Ebong, 1981). However, contrary to contemporary national leveling networks, the Nigerian VRF consists of only "spirit leveled elevation differences" that have not been corrected for the effect of gravity hence the Normal Orthometric Height (NOH) has been adopted for vertical control points (Ebong *et al.*, 1991) rather than true Helmert Orthometric Height (HOH). This seeming uncertainty about the nationally adopted height system in use and exacerbated by the difficulties associated with spirit leveling, has led to the use of multiple height systems across the country (Isioye *et al.,* 2010; Badejo *et al.*, 2016). In reality, most topographic surveys and by extension associated engineering projects are constructed by referring to arbitrary height datum rather than using orthometric heights. This situation has led to the inability of local authorities to determine easily (with the use of client's topographic plan) the relative undulation between proposed sites and nearby infrastructure. As such, building approvals have been granted in flood prone areas which could have been easily detected if actual orthometric heights are utilized on the submitted topographic maps. Furthermore, the absence of a properly defined VRS has had a negative toll on the production of bathymetric and navigational charts in Nigeria. Therefore, the need for an urgent adoption of a national height system cannot be over emphasized. Isioye *et al*. (2010) evaluated the use of normal gravity instead of observed gravity and its distortion on the leveling network. He discovered that the use of Normal heights instead of Normal Orthometric Heights showed statistically no significant difference between both height systems on the examined benchmarks. However, the study did not examine the effect of the use of NOH instead of HOH within the studied network. Odumosu *et al.* (2018) examined the statistical implications of replacing HOH with NOH and in his study discovered that there is no significant difference between both systems within the lowland area of Lagos state. Researchers such as Steinberg and Papo (1998), Kumar (2005) and Badejo *et al*. (2016) have supported the use of Ellipsoidal heights instead of orthometric heights but limit the application of such replacement to airborne mapping applications, marine navigation and the low land areas only. However, further study by Eteje *et al*. (2018) recommends that rather than absolute reliance on ellipsoidal heights, the local geoid of the study area be used to convert ellipsoidal heights to Orthometric heights. This study however, presents practical/computational options for the Nigerian geodetic community for the re-definition of our VRS by adopting either the

NOH or the HOH given spirit leveled elevation differences, absolute gravity and ellipsoidal heights of some benchmarks across Lagos state.

## 2.0 Height Systems

In this section, the Helmert Orthometric Height, Normal Orthometric Height and Ellipsoidal Height are discussed in brief with a view to identifying their theoretical implications, reference surfaces and computational evaluation. The discussion is limited to these three systems because they are the three major systems that are of concern to this study considering the Nigerian scenario.

*Orthometric Height System*:

The Orthometric height ($H^{ortho}$ in Figure 1) is defined as the length of the curved plumbline from a point "$P$" (on the earth surface) to its intersection with the geoid at point "$P_0$" (Fotopoulos, 2003; Matt, 2010). The Orthometric height is mathematically described as given in equation (1).

$$H^{Ortho} = \frac{C}{g_{mean}} \qquad (1)$$

Where:
$H^{Ortho}$ = Orthometric Height
$C$ = Geopotential Number
$g_{mean}$ = Mean Value of the actual gravity along the curved surface of the plumbline between the geoid and the topographic surface. It is mathematically described as given in equation (2)

$$g_{mean} = \frac{1}{H_0}\int_{P_0}^{P} g\, dH \qquad (2)$$

Where:
$H_0$ = Approximate value of the Orthometric Height
$P$ = Point on the earth surface
$P_0$ = Corresponding point on the geoid
$g$ = Measured gravity value at points along the plumb line
$dH$ = Differential elements along the plumb line between the geoid and the point on the earth surface

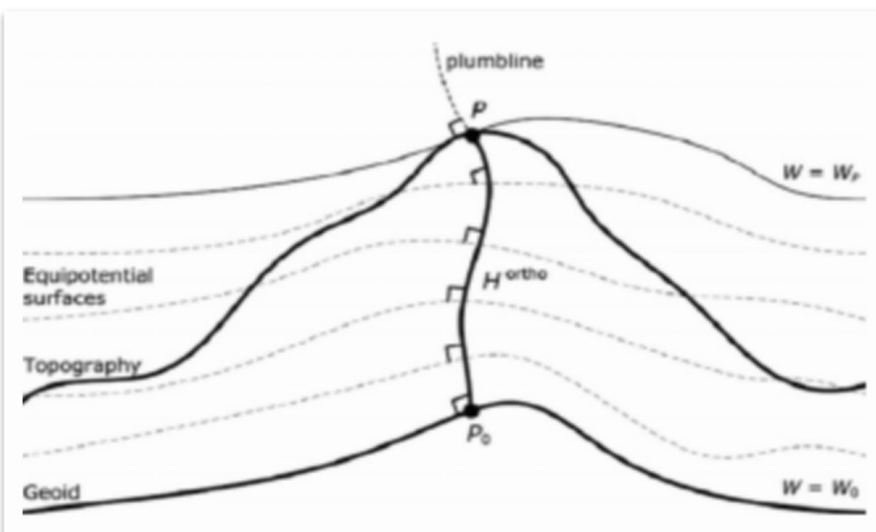


**Figure 1:** The orthometric height (Adapted from Featherstone and Kuhn, 2006)

Practically, in order to overcome the complexities and rigors involved in evaluating equation (2), Orthometric heights can be obtained by applying requisite corrections for the equi-potential surfaces on the measured spirit leveled elevation differences (Heskanen and Moritz, 1967). This correction commonly called the "Orthometric correction" requires that gravity or its variant be measured at the level stations. Due to the complexities involved in the determination of gravity for all points within a leveling network, several variants of the orthometric height system exist with each variant differing by accuracy and computational reliability from others. In Nigeria, the NOH system (depicted as ($H^{N-o}$) in Figure 2) was used for most of the SBM's and FBM's as against the HOH due to lack of gravity values at the leveling stations.In the NOH, the actual gravity values of the leveling stations are replaced with the use of normal gravity values. Although, this is the practice in most countries including Australia (Featherstone and Kuhn, 2006); this replacement mathematically eliminates the physical meaning of the NOH, thus making it only a geometrically meaningful height system (Featherstone and Kuhn, 2006). Furthermore, replacing actual gravity with the normal gravity value in the evaluation of the NOH mathematically changes the assumption of non-parallelism of the equi-potential surfaces of the gravity field to that of the sphero-potential surfaces of the normal gravity field (Heck, 2003; Tenzer *et al.*, 2005). By implication, the NOH system may not be efficient for prediction of fluid flow in areas with very rough topography and may result in a situation where water flows uphill.

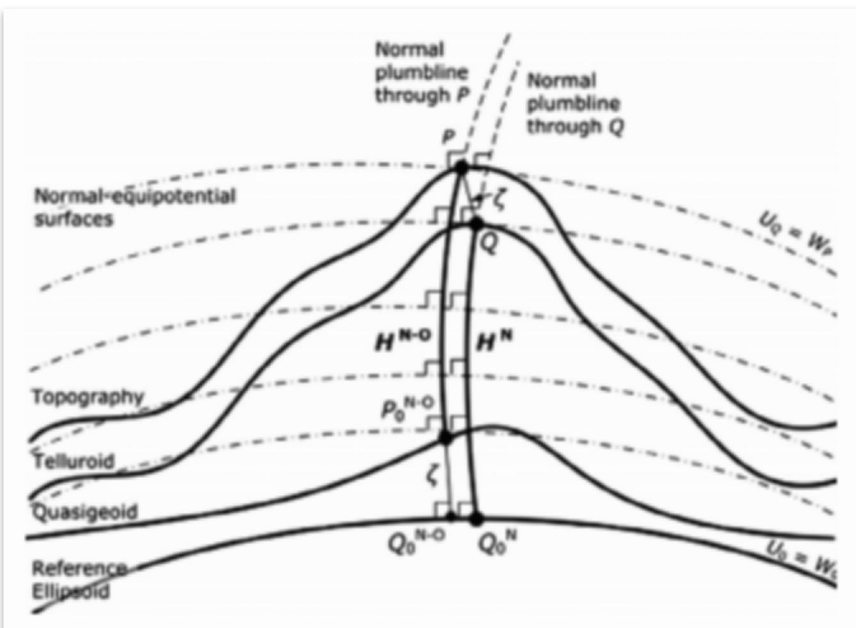


**Figure 2:** The normal and normal-orthometric heights (Adapted from Featherstone and Kuhn, 2006)

Depending on available data, both the HOH and the NOH could be obtained by application of either the Orthometric correction ($OC$) or the Normal Orthometric correction ( $NOC$ ) respectively to spirit leveled elevation differences.

The mathematical formulae for OC and NOC are given in equations (3) and (4) respectively.

$$OC_{AB} = \sum_A^B \frac{g-\gamma_0}{\gamma_0}\,\delta n + \frac{\bar{g}_A-\gamma_0}{\gamma_0} H^*{}_A - \frac{\bar{g}_B-\gamma_0}{\gamma_0} H^*{}_{\boldsymbol{B}} \qquad (3)$$

Where:

$\delta n$ = measured elevation difference between stations A and B

$g$ = Actual gravity measured along the route

$\overline{g}_A$ and $\overline{g}_B$ = actual average gravity at stations A and B

$H^*{}_A$ and $H^*{}_{\boldsymbol{B}}$ = known spirit leveled elevation at stations A and B

$\gamma_0$ = A constant value usually the normal gravity at geographic latitude 45°

$$NOC = \sum_A^B \frac{g-\gamma_0}{\gamma_0}\,\delta n + \frac{\gamma_A-\gamma_0}{\gamma_0} H^*{}_A - \frac{\gamma_B-\gamma_0}{\gamma_0} H^*{}_{\boldsymbol{B}} \qquad (4)$$

Where:

$\delta n$ = measured elevation difference between stations A and B

$g$ = Actual gravity measured along the route

$\overline{g}_A$ and $\overline{g}_B$ = actual average gravity at stations A and B

$H^*{}_A$ and $H^*{}_{\boldsymbol{B}}$ = known spirit leveled elevation at stations A and B

$\gamma_0$ = A constant value usually the normal gravity at geographic latitude 45°

$\gamma$ = normal gravity at leveling station as given in equation (5)

$$\gamma = 978\,031.85\,(1 + 0.00527889 \sin^2\varphi + 0.000023462\, sin^4\varphi) \qquad (5)$$

Where

$\varphi$ = geographic latitude of leveling station

*Ellipsoidal Height System*

The Ellipsoidal height depicts the vertical separation between the reference ellipsoid and the topographic surface measured along the direction of the normal to the ellipsoid as shown in Figure 3. Ellipsoidal heights are mathematical easy to calculate due to the fact that the ellipsoid is a mathematically closed figure. Consequently, the Ellipsoidal height does not have any physical meaning but purely a geometrical meaning and as such is not suitable for prediction of fluid flows.

With the advent of GNSS receivers, determination of ellipsoidal heights of points on the earth surface has become a fast, less cumbersome and computationally simple task compared to spirit leveling operation that is required for determination of Orthometric heights. This has recently popularized the use of ellipsoidal heights in place of orthometric heights in several engineering works. Although, earlier studies by Badejo *et al.* (2011) revealed that the use of ellipsoidal heights instead of orthometric heights do not have effect on the resulting topographic configuration in the low land area of Port Harcourt and Lagos state, the desire for a unified height system would require that a reliable gravimetric geoid model within the study area is applied to measured GNSS ellipsoidal heights in order to convert them to Helmert Orthometric heights using equation (6) as presented by Dinter *et al.* (2001).

$$N = h - H \qquad (6)$$

Where

$N$ = Geoid undulation (Geoid model)

$h$ = Ellipsoidal Height

$H$ = Orthometric Height

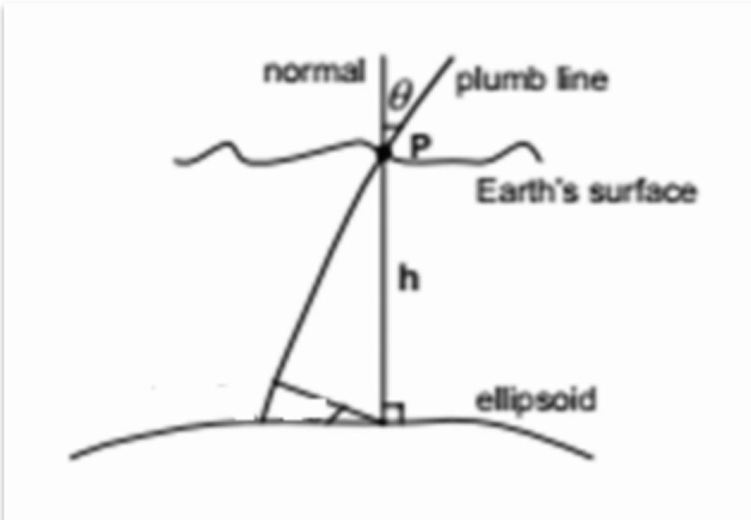


**Figure 3:** Graphical description of the ellipsoidal height (Hofmann-Wellenhof and Moritz, 2006)

### 3.0 Available Options

Three height systems have been discussed in Section 2.0 with each method having its advantages and disadvantages over the other. The computational formulae for the conversion of these field measurements (either spirit leveling or GNSS measurements) to the standard HOH or the NOH have also been presented in Equations (3)-(6). Table 1 presents a summary of the practical options available for Nigeria for a national redefinition of our adopted VRS. In the table, advantages and disadvantages of each option is presented with emphasis on the scientific and by extension practical implication of each option on national height policy.

Amongst others, a major disadvantage of the present-day use of multiple height systems is the problem of the compatibility of datasets, where lay users may incorrectly integrate ellipsoidal heights with other types of height (Featherstone and Kuhn, 2006). Also, the use of multiple height systems, as earlier mentioned, increases the risk of flooding and inundation within localities; as different topographic maps of adjacent land parcels may not be useful for interpreting fluid flow across both parcels due to inconsistent height types used for mapping the individual land parcels by the different map makers.

**Table 1:** Practical Options for the Re-definition of the Nigerian VRS

| Field Measurement | Adopted Height System / Computational approach | Advantage | Disadvantage |
|---|---|---|---|
| Spirit Leveling | HelmertOrthometric Heights (HOH) / Orthometric Correction (OC) | 1. A physically and geometrically meaningful height system<br>2. Origin of measurements is located at the geoid (approximately Mean Sea Level)<br>3. Suitable system for prediction of fluid flows | 1. The need to measure the absolute gravity value at leveling stations is costly and time-consuming<br>2. Precise determination of the gravity vector along the plumb line is a highly subjective and route-dependent observational scheme<br>3. Satisfying the requirement of the first term of the OC requires a carefully planned field observation procedure |
| Spirit Leveling | Normal Orthometric Heights (NOH) / Normal Orthometric COrrection (NOC) | 1. Replaces the need for mean actual gravity at leveling stations with mean normal gravity<br>2. Origin of measurements is located at the quasi-geoid; a near-surface to the geoid<br>3. Consequent upon (2), NOH can suitably be used to replace HOH in areas with near-even topography (Odumosu *et al.*, 2018) | 1. By definition, the replacement of geo-potential number with the sphero-potential number (Heck, 2003) reduces the height system to a geometrical system<br>2. Consequent upon (1) replacement of actual gravity with normal gravity makes the system lose its physical meaning<br>3. Since actual gravity deviates largely from normal gravity in highly mountainous areas, it is theoretically assumed that NOH be limited in application to lowlands |
| Ellipsoidal Heights | Helmert Orthometric Heights (HOH) / Geoid Model | 1. Acquiring ellipsoidal heights via GNSS is fast and less tedious<br>2. Not much skill is required for data capture in the field once the appropriate parameter settings have been done<br>3. All other advantages as stated in option (1) above | 1. Due to a one-sided GDOP, obtaining 3rd order accuracy ellipsoidal heights from GNSS receivers requires static observation with not less than 20mins occupation time.<br>2. Heights measured by faster techniques are subject to measurement jumps that could be as high as 1m in unchecked situations<br>3. Consequently, more time is taken at a leveling station with GNSS technique |

## 4.0 Case Study

A case study of the proposed options has been conducted within Lagos state. Lagos State is a Low-lying coastal state having a fairly stable terrain with minimal undulation and an approximate landmass area of about 3600 Sq km. Bounded in the South by the Atlantic Ocean and the Lagoon; several other tributaries from the Lagoon extend into the state some of which include the five cowries, the Iddo Port, Apapa port amongst others.Being the host state where the nation's vertical datum (Apapa Datum) is established, most control points within the state have spirit leveled elevation values observed on them. Figure 4 shows the bench marks used for this study and their spatial spread across Lagos state. As shown in the legend, the black dots represent bench marks upon which gravity observation was taken (see Odumosu ***et al***, 2017).

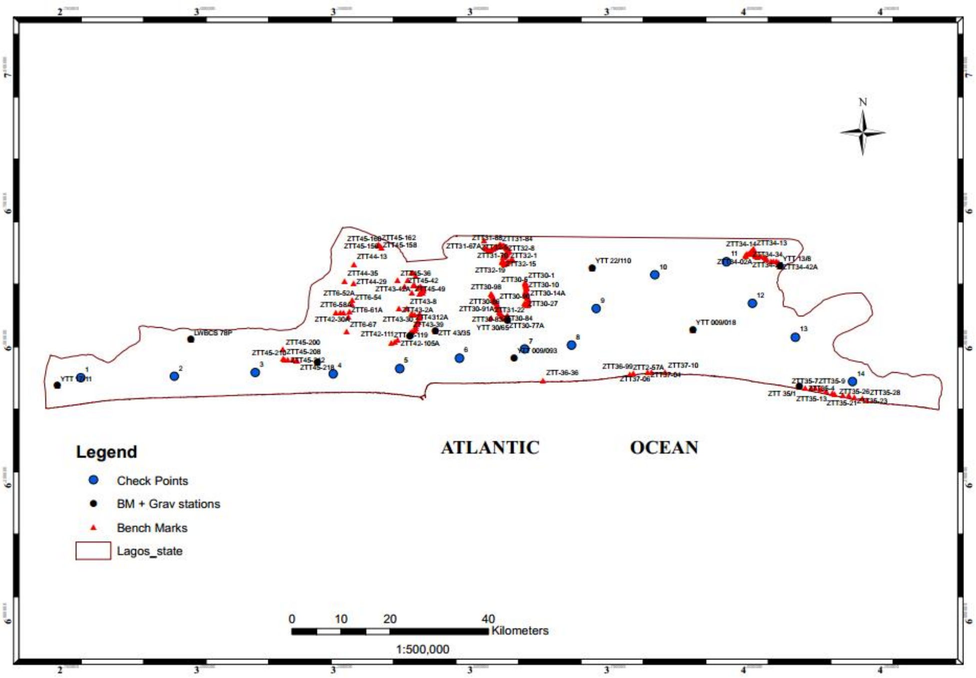


**Figure 4**: Study area and spatial distribution of bench marks used for the study

## 5.0 Materials and Methods

A total of 218 bench marks were used in this study. Amongst the 218 bench marks, precise gravity observation was carried out on 10 stations using a Scrintrex CG4 gravimeter in a step observation method. A brief description of the data used is given in Table 2 while a description of the methods is given in Tables 3(a)-(c). Full description of the approach summarized in Table 3 can be found in Odumosu *et al.* (2017) and Odumosu (2019). In order to facilitate easy check of the results, 14 of the stations were selected as check points across the study area. The essence of selecting points at different locations across the state is to ensure that the effects of non-parallelism of the equi-potential surfaces are put into consideration across the study area. The 14 check points are shown in blue bigger dots in Figure 4.

**Table 2:** Data used

| S/N | Data type | Source | Observational Procedure | Accuracy |
|---|---|---|---|---|
| 1 | Elevation | Office of the Surveyor General of Lagos State | Spirit Levelling | 2nd Order |
| 2 | Ellipsoidal Height | Office of the Surveyor General of Lagos State | Static GNSS Observations | 2nd Order |
| 3 | Absolute Gravity | Primary data collection for 10 points | Step method of observation | 0.1mgals |
| | | LSC predicted for 208 points | N/A | 0.48mgals |
| 4 | Gravimetric Geoid of Lagos State | The South Western Gravimetric Geoid 2018 (SWGG018) | Description of computational approaches can be found in Odumosu (2019) | 26.6 cm |

**Table 3(a):** Description of observational procedure for gravity measurement

| S/N | Parameter | Value |
|---|---|---|
| 1 | Drift rate | 0.0007mgals/sec |
| 2 | Observational Method | Step method |
| 3 | Equipment used | Scrintrex CG5 |
| 4 | No of Days | 3days |
| 5 | Control point | 03663L (MuritalaMohd. Airport) |

**Table 3(b):** Description of computational procedure for gravity prediction

| S/N | Parameter | Value |
|---|---|---|
| 1 | Prediction Method | Least Squares Collocation (LSC) |
| 2 | Covariance Estimation Method | Marquardt Lavenberg Method (NLP approach) |
| 3 | Validation Approach | Field observation and Leave Out Validation (LO) Methods |
| 4 | Base Data | Bureau Gravimetrique International (BGI) datasets across Nigeria<br>Nigerian Gravity Standardization Network (NGSN 84) data (Osazuwa, 1984) |

**Table 3(c):** Description of computational procedure for the SWGG'018

| S/N | Parameter | Value |
|---|---|---|
| 1 | Total number of gravity points | 2148 points |
| 2 | Computational Approach | Remove Restore Compute |
| 3 | Long wavelength Geoid | EGM96 (Best fit global geoid model within the study area) |
| 4 | Stokes computational approach | Simpson's one-third rule for double integral computation |
| 5 | Terrain Correction (TC) | Hammer Chart correction method with an inner and outer ring distance of 1km and 166km respectively at constant density of $2670\ kg\ m^{-3}$ |

## 6.0 Results

After all relevant computations as earlier discussed, the HOH and NOH across the study area was computed and the graphical model is as presented in Figures 5(a) and (b). Also, the SWGG'018 was applied to the given ellipsoidal heights of the bench marks to derive the Helmert Orthometric heights. Extracts of the obtained values for each of the computed Height system options at the 14 check points randomly selected across the study area is as presented in Table 4.

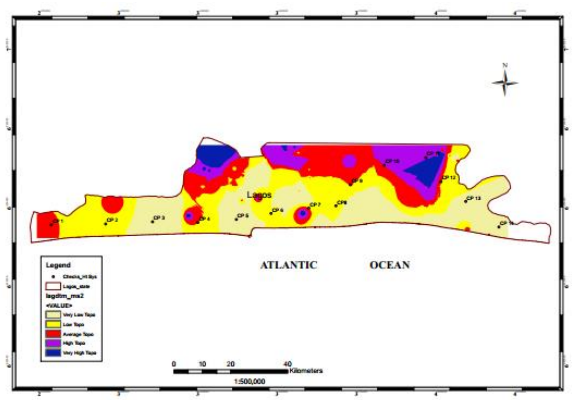


**Figure 5(a):** Normal orthometric heights (NOH)

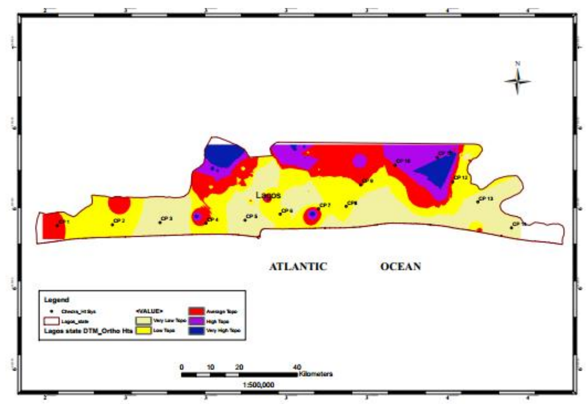


**Figure 5(b):** Helmert orthometric heights (HOH)

**Table 4**: Extract of Results at 14 check points located across the study area

| Check Point | HOH Option 1 | NOH Option 2 | EllipHt | Grav Geoid | Derived HOH Option 3 |
|---|---|---|---|---|---|
| CP 1 | 23.4874 | 23.5465 | 46.3111 | 22.8117 | 23.4994 |
| CP 2 | 17.8694 | 17.8939 | 40.6131 | 22.6740 | 17.9390 |
| CP 3 | 8.1646 | 8.1920 | 30.7303 | 22.5346 | 8.1956 |
| CP 4 | 22.0544 | 22.1095 | 44.4758 | 22.3693 | 22.1064 |
| CP 5 | 8.3635 | 8.4087 | 30.6760 | 22.4015 | 8.2745 |
| CP 6 | 16.6970 | 16.7708 | 39.2747 | 22.4811 | 16.7936 |
| CP 7 | 20.8143 | 20.9036 | 43.3902 | 22.4999 | 20.8903 |
| CP 8 | 18.6372 | 18.6853 | 41.2061 | 22.5479 | 18.6582 |
| CP 9 | 21.5956 | 21.6351 | 44.5023 | 22.9757 | 21.5266 |
| CP 10 | 31.1245 | 31.1768 | 54.2068 | 23.1073 | 31.0995 |
| CP 11 | 39.7374 | 39.8314 | 62.7400 | 22.9816 | 39.7584 |
| CP 12 | 21.4378 | 21.4879 | 44.6227 | 23.1719 | 21.4508 |
| CP 13 | 10.6688 | 10.6951 | 33.7509 | 23.0671 | 10.6838 |
| CP 14 | 4.8747 | 4.8928 | 28.0881 | 23.1914 | 4.8967 |

Descriptive statistics of the values obtained from the HOH and NOH are presented in Table 5. The difference in standard deviation between both systems is 1.4 cm while the differences between the maximum and minimum heights obtained from both systems are 10 mm and 9 mm, respectively. These centimetre-level differences are consistent with the magnitudes reported for low-relief terrains in similar comparative studies (Featherstone & Kuhn, 2006; Eteje et al., 2018; Odumosu et al., 2018). Theoretically, such close agreement is expected because, in regions of mild topography, the deflection of the actual gravity field from the normal gravity field is minimal, and the spheropotential surfaces approximate the geopotential surfaces with sub-decimetre accuracy (Heck, 2003; Tenzer et al., 2005). Consequently, the residual error introduced by replacing measured gravity with normal gravity in the orthometric correction is largely absorbed within the noise level of the spirit-leveled observations themselves (Featherstone & Kuhn, 2006). This statistic therefore suggests that a very strong similarity with an inconsequential difference exists between both height systems for the study area. To formally establish whether the apparent similarity between the two systems is statistically significant, a Student's "t" test of significant difference between the means of both systems was carried out at the 95% confidence level, as shown in Table 6.

**Table 5:** Descriptive statistics of the HOH and NOH for 218 bench marks within the study area

| | **Helmert Orthometric Height (HOH)** | **Normal Orthometric Height (NOH)** | **Diff** |
|---|---|---|---|
| **Mean** | 21.75014898 | 21.80631976 | -0.05617 |
| **Standard Error** | 0.960156056 | 0.961140393 | |
| **Median** | 20.07110915 | 20.12450867 | |
| **Standard Deviation** | 14.17653428 | 14.19106784 | -0.014534 |
| **Sample Variance** | 200.9741241 | 201.3864064 | -0.41228 |
| **Kurtosis** | -1.091097056 | -1.089331621 | |
| **Skewness** | 0.262580208 | 0.263451063 | |
| **Range** | 51.95544214 | 51.96042141 | -0.00498 |
| **Minimum** | 1.195556865 | 1.200131164 | -0.00457 |
| **Maximum** | 53.15099901 | 53.16055257 | -0.00955 |
| **Sum** | 4741.532477 | 4753.777708 | |
| **Count** | 218 | 218 | |

**Table 6:** Student's "t" statistics to show statistically significant difference between the means of the HOH and NOH within the study area

| | **H O H** | **N O H** |
|---|---|---|
| **Mean** | 21.75014898 | 21.80631976 |
| **Variance** | 200.9741241 | 201.3864064 |
| **Observations** | 218 | 218 |
| **Pearson Correlation** | 0.999997201 | |
| **Hypothesized Mean Difference** | 0 | |
| **df** | 217 | |
| **t Stat** | -22.67850978 | |
| **P(T<=t) one-tail** | 0.00 | |
| **t Critical one-tail** | 1.651905861 | |

With a Pearson correlation value of 0.9999, it is obvious that there is a very strong correlation between the HOH and the NOH. This is further confirmed with the "t-critical" value exceeding the "t-statistics" value. It is therefore concluded that there is no statistical difference between the HOH and the NOH within the study area. The obtained result conforms to earlier findings by Odumosu *et al.* (2018) and Isioye *et al.* (2010), and is in agreement with the broader theoretical position of Featherstone and Kuhn (2006) that NOH is an acceptable substitute for HOH in low-relief terrain. Comparison of Figures 5(a) and (b) indicates that there is no physical difference in the spatial distribution of heights across the study area as produced using the HOH and the NOH systems. This therefore authenticates the scientific rationale behind the use of NOH instead of HOH in low lands with mild terrain undulation, as previously established for similar low-altitude regions (Featherstone & Kuhn, 2006; Odumosu et al., 2018). A plot of the profile using both height systems across the check points chosen within the study area is presented in Figure 6, while the differences between both height systems at the check points is graphically shown in Figure 7. The systematic offset (NOH > HOH) observed in the profile is consistent with the structural relationship between the spheropotential and geopotential surfaces in coastal lowlands described by Heck (2003) and Tenzer et al. (2005), and similar low-magnitude offsets have been reported in other low-altitude harmonization studies (Eteje et al., 2018; Odumosu et al., 2018).

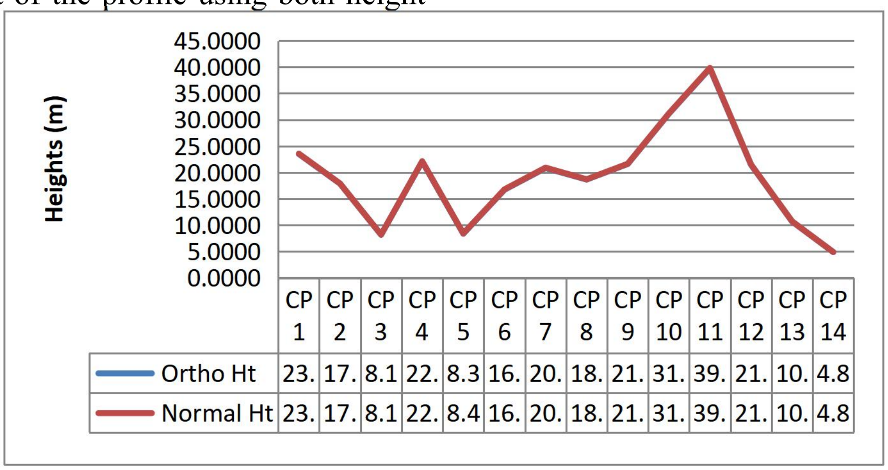


**Figure 6:** Profile plan along check points using HOH and NOH

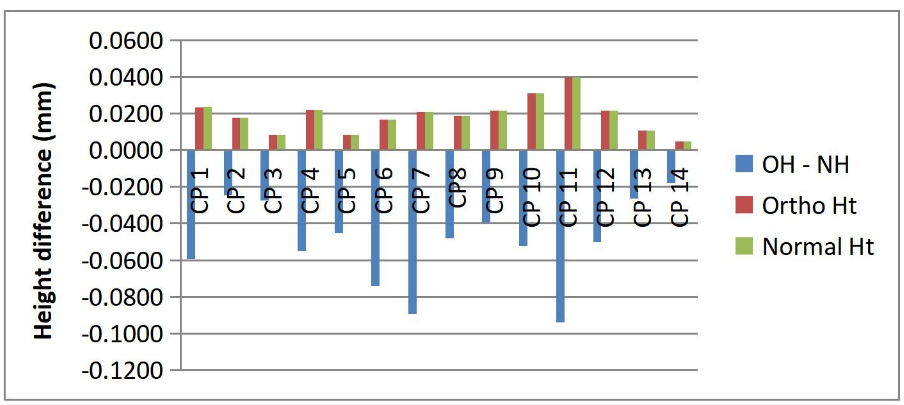


**Figure 7:** Differences between the HOH and NOH at the check points

From Figures 6 and 7, the maximum difference between the HOH and NOH across the check points is 9cm. Table 7 presents the differences at the check points between the computed HOH and the derived HOH at the check points

**Table 7:** Differences between computed HOH

| Sta ID | Derived HOH (Option 3) | HOH (Option 1) | Diff (m) |
|---|---|---|---|
| CP 1 | 23.4994 | 23.4874 | -0.01 |
| CP 2 | 17.9390 | 17.8694 | -0.07 |
| CP 3 | 8.1956 | 8.1646 | -0.03 |
| CP 4 | 22.1064 | 22.0544 | -0.05 |
| CP 5 | 8.2745 | 8.3635 | 0.09 |
| CP 6 | 16.7936 | 16.6970 | -0.10 |
| CP 7 | 20.8903 | 20.8143 | -0.08 |
| CP 8 | 18.6582 | 18.6372 | -0.02 |
| CP 9 | 21.5266 | 21.5956 | 0.07 |
| CP 10 | 31.0995 | 31.1245 | 0.02 |
| CP 11 | 39.7584 | 39.7374 | -0.02 |
| CP 12 | 21.4508 | 21.4378 | -0.01 |
| CP 13 | 10.6838 | 10.6688 | -0.02 |
| CP 14 | 4.8967 | 4.8747 | -0.02 |

Similarly, from Table 7, the maximum difference between the computed HOH using option 1 and the gravimetric geoid-derived HOH using option 3 is 10 cm. This shows a similarly strong correlation between the derived HOH using option 3 (as suggested in section 3.0) and the computed HOH using option 1 (as suggested in section 3.0). The 10 cm spread is comparable to the 7–26 cm geoid agreement reported for similar gravimetric/GNSS-leveling integration studies in the Niger Delta and Benin City (Eteje et al., 2018; Odumosu, 2019), and is well within the 3rd-order leveling tolerance recommended by Surveyors Council of Nigeria specifications. This implies that, within the test area, the SWGG'018-derived HOH from GNSS ellipsoidal heights is metrologically equivalent to the gravity-corrected HOH from spirit leveling, supporting the practical adoption of option 3 for the redefinition of the Nigerian VRS in low-relief regions.

## 7.0 Conclusion

Three solution approaches for determination of a consistent Height system for Nigeria has been presented in this study using the low lands of Lagos state as a case study. The first option is to compute the Helmert Orthometric Heights (HOH) of leveling stations using measured gravity and the spirit leveled elevation difference at the station. The merits and demerits of this option have been evaluated in the study and the several ambiguities associated with gravity measurements have been identified as the major de-merit of the option. The second option as suggested in this study is to convert the spirit leveled elevation differences to Normal Orthometric Heights since normal gravity of the leveling stations can easily be computed without the need for actual gravity observations. However, because of the implications of replacing actual gravity with normal gravity at the measured stations, the height system is merely geometrical and has no physical meaning. As such, this option although suitable in the tested low land area of

Lagos state may not be suitable for prediction of fluid flow in hilly and rugged terrain. Finally, the third option presented is to convert the ellipsoidal heights which are easier to obtain from GNSS observations to Helmert Orthometric Heights using a reliable gravimetric geoid model of the area. Although, this method requires gravity measurements for determination of the gravimetric geoid, it does not require subsequent measurement of gravity value at the leveling stations. Therefore, this method by implication also produces a physically and geometrically meaningful height system that is suitable for prediction of fluid flow even in very hilly and rugged areas. From the analysis of the results as presented in section 6.0, it is obvious that there exists no significant difference between the three options in the low land area of Lagos state. However, considering the computational and observational ease involved in all the three options alongside with their earlier discussed merits and de-merits, option 3 is considered the most efficient option for the re-definition of the Nigerian VRS.

## 8.0 Recommendations

Based on the outcomes of the study, the following recommendations are proposed:

1. Precise gravity observations should be conducted at recommended intervals across the country in preparation for the computation of an accurate national gravimetric geoid that can be used for converting ellipsoidal heights to Orthometric heights nationwide.
2. Meanwhile, the already established South West Gravimetric Geoid (SWGG'018) developed by Odumosu (2019) could be officially adopted for the South Western geo-political zone of Nigeria. Similarly, a committee could be established by SURCON in conjunction with OSGOF, NIS and the academia to look into the several researches conducted in geodesy departments across Nigeria on geoid modeling. The intent of the committee should be to harmonize with a view to integrating the concepts, approaches and data used by the different researchers towards establishment of a tentative national geoid pending when recommendation one above can be achieved.
3. The surveying community should advocate for legislation on the acceptable height system upon which all topographic maps, charts and all other height related works should be presented. Such legislation would save the nation from several impending engineering and construction related disasters especially as flooding events are expected to increase in response to the increasing rate of sea level rise and global warming.

## Declaration of Competing Interests

The authors have no conflict of interest to declare that are relevant to this article

## Data Availability

Data will be available on request.

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